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Mathematical Philosophy of Takebe Katahiro

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Abstract

Takebe Katahiro (1664-1739) was a Japanese mathematician, who exposed his philosophy on Mathematics itself and on Mathematical Research.

In the *Taisei Sankei* (1711), he wrote Chapter 4 entitled the "Three Essentials" describing four classes of mathematical problems, the status of parameters in a problem and the classification of numbers. Note that his thought was based on Chinese traditional philosophy and on the achievement of Japanese mathematics in the early Edo period. Note also he recognized some numbers are not algebraic.

In the *Tetsujutsu Sankei* (1722) he recommended the inductive heuristic method in mathematical research and recognized a mathematical research would be successful once the character of a mathematical object and that of a mathematician were accommodated to each other, comparing his method of calculation of the circular coefficient with that of his master Seki Takakazu and citing his own discovery of the infinite power series expansion formula of an inverse trigonometric function.

The Meiji Restoration (1868) was a turning point in the history of Japan from the feudalism to the constitutional monarchism. The "ordinance on school system" (1872) of the new government defined mathematics in Japanese schools to be of "European style" thus abandoning the Japanese traditional mathematics. This policy was proved to be efficient at least for a half century and Takebe's philosophy on Mathematics was buried in the complete oblivion.

1 Takebe Katahiro

Takebe Katahiro 建部賢弘 (1664-1739) was one of great mathematicians in the Edo Period (1603-1868) of Japan. When he was 13 years old, he became a student of Seki Takakazu 関孝和 (ca. 1642-1708) in 1676. He was a precocious mathematician and published his first monograph, the *Mathematical Methods for Clarifying Slight Signs* 研幾算法 *Kenki Sanpō* (1683) when he was 20 years old. He was a faithful student and published the *Colloquial Commentary on the Operations* 演段諺解 *Endan Genkai* (1685), an elaborate commentary on his master's abstruse monograph, the *Mathematical Methods for Exploring Subtle Points* 発微算法 *Hatsubi Sanpō* (1674). In the *Colloquial Commentary* he developed a new method for eliminating variables from a system of simultaneous algebraic equations of several variables in order to obtain an equation of one variable, which can be solved numerically by a Chinese procedure of the "celestial element." He was diligent and studied a Chinese mathematical monograph, the *Introduction to Mathematics* 算学啓蒙 *Suanxue Qimeng* (1299) of Zhu Shijie 朱世傑 and reprinted it with plenty of comments as the *Complete Colloqial Commentary* 諺解大成 *Genkai Taisei* (1690). He realized, although much more advanced than its Chinese predecessor, their mathematics had its foundation in the Chinese mathematics of Song Dynasty.

According to the *Biography of the Takebe* 建部氏伝記 *Takebe-shi Denki*, Seki Takakazu and Takebe brothers (Kataakira 賢明 and Katahiro) complied the *Complete Book of Mathematics* 大成算経 *Taisei Sankei*, an encyclopedic mathematical monograph of 20 volumes, during the

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period of 1683 - 1711. The Complete Book was so voluminous that it could not be printed; only a few hand written manuscripts were left to posterity. In 1722 he wrote the *Mathematical Treatise on the Technique of Linkage* 綴術算経 *Tetsujutsu Sankei* and dedicated it to the 8th shōgun Yoshimune. This book was very handsome and circulating well among students of the Seki's school. There are many versions of it, which are transmitted to today.

Katahiro was not a professional mathematician in today's sense. In 1692 he became a retainer of Lord of Kōfu, who became the 6th shōgun in 1709. Following his lord, Katahiro moved to Edo (today's Tokyo) and engaged mainly in governmental duties in the Tokugawa shōgunate. As a competent officer he served three shōgun until his retirement in 1733. One of his achievements as governmental officer was the preparation of the *Atlas of Japan* 国絵図 *Kuni Ezu* (1725). See [6] for details.

Katahiro was a rare Japanese mathematician who exposed on philosophy of mathematics and mathematical research. There are two instances; Volume 4 of the Complete Book of Mathematics (1711) and the Technique of Linkage (1722). We shall introduce them one by one.

2 The *Taisei Sankei*

The *Complete Book of Mathematics* (the *Taisei Sankei*) was complied by Seki Takakazu and the Takebe brothers (Kataakira and Katahiro) during the period of 1683-1711 (28 years) according to the Biography of the Takebe. (See [3], p. 270 and [8], Chapter 2.) At the final stage of compilation as Seki Takakazu was senile and Katahiro was busy as government officer, Katahiro's elder brother Kataakira completed the task alone. The Book contained all mathematics known to them, especially new theories inaugurated by Seki Takakazu.

2.1 Contents

Let us introduce its contents briefly.

The 20 volumes Book is fowarded by Introduction 首篇, which includes Discussion on Mathematics and Numbers 算数論, Basic Numbers 基数, Large Numbers 大数, Small Numbers 小数, Degree 度数, Quantity 量数, Weight 衡数, Time 鈔数,Counting Board 縦横, Red and Black Counting Rods 正負, Operation on Counting Board 上退, and Terminologies 用字例.

The 20 volumes are divided into three Parts, Part A, B, and C.

Part A (Volumes 1 to 3) treats elementary arithmetic ending with the introduction of determinants. Volume 1 is entitled Five Techniques 五技 and treats Adition 加, Subtraction 減, Multiplication 因乗, Division 帰徐, and Extraction of Root 開方; Volume 2 is entitled Miscellaneous Techniques 雑技 and treats Addition and Subtraction 加減, Multiplication and Division 乗除, and Extraction of Root 開方; and Volume 3 is entitled Various Techniques 変技 and treats advanced aspects of the contnet of the previous volume.

Part B (Volumes 4 to 15) can be further divided into three blocks; Volume 4 serves an intoroduction of Part B, Volumes 5 to 9 treat traditional mathematics and games, and Volumes 10 to 15 treat various problems on geometry and measurement.

Volume 4 is named Three Essentials 三要 and includes Symbol and Figure 象形, Flow and Ebb 満干, and Numbers 数; Symbol and Figure are the classification of mathematical objects and hence, problems. We shall discuss the Three Essentials later.

Volumes 5 to 9 are named Method of Symbol 象法 and discuss problems on "symbols." Volume 5 treats Mutual Multiplication 互乗, Repeated Multiplication 畳乗, and Pile sums 垜積; Volume 6 treats Fractions 之分, Several Methods of fractions 諸約, and Art of Cutting Bamboo 翦管; Volume 7 treats Magic Squares, Magic Circles 聚数, Josephus Problems 計子, 算脱, Coding Problems 験符; and Volumes 8 and 9 treat Daily Mathematics 日用術.

Volumes 10 to 15 are named Method of Figure 形法 and discuss problems on "figures". Volume 10 treats Regular Squares 方, Rectangles 直, and Regular Triangles 勾股, Polygons 斜 (三斜、四斜、五斜); Volume 11 discusses Regular Polygons 角法; Volume 12 is concerned with Coefficients of Figure 形率, i. e., Circle Theory 円理, and treats the length of the circular circumstance 円率, the length of an arc 弧率, the volume of a ball 立円率, and the volume of spherical figures 球闕率. Volume 13 is the same as Seki Takakazu's monograph the Measurement 求積. Volumes 14 and 15 are concerned with Techniques of Figure 形巧.

Part C (Volumes 16 to 20) treats Seki's theory of equations. Volume 16 is named Discussion on Problems and Procedures 題術辨 and the same as Seki's monograph the *Critical Studies of Problems* 題術辨議之法. It serves as an introduction to Part C. Volume 17 is named Solutions of All Problems 全題解 and similar to Seki's monograph the *Trilogy* 三部抄, which contains Explicit Problems (i.e., direct calculation) 見題, Implicit Problems (i.e., equation of one variable) 隠題, Concealed Problems (i.e., equation of several variables) 伏題, and Submerged Problems (i.e., non algebraic equations) 潜題. In solving concealed problems, Seki discovered formulas for resultants and diterminants. Volume 18 is similar to Seki's monograph *Restoration of Defective Problems* 病題擬; and Volumes 19 and 20 are named Examples of Operations 演段例 and contain 23 examples of algebraic equations.

2.2 Three Essentials

Along with the tradition of Chinese mathematics, Takebe Katahiro recognized mathematics as a bunch of mathematical problems. He tried to classify mathematics (i.e., mathematical problems) and to organize the Complete Book of Mathematics. Volume 4 was named the Three Essentials 三要, in which Takebe Katahiro's philosophy on mathematics was exposed. The Three Essentials are divided into three sections: Symbols and Figures 象形; Flow and Ebb 満干; and Numbers 数. Each section starts with a general statement followed by problems (67 in total) which serve as examples for the general statement.

Three Essentials starts with the following introductory statements:

> All mathematics [problems] are originated from symbols and figures, which are the beginning of a problem, have a determined formula and vary according to occasions. Nevertheless, as there are ways to change the flow and ebb, numbers are useful for solving the problem[2]. These three essentials are the basis of all the mathematical investigations[3]. Certainly, a theory is equipped and numbers are involved in all aspects of mathematics starting from technique of problem solving to the movement of the heaven and the earth[4]. Understanding this principle, students should observe all the changes of a thing to investigate its theory [5].

The authors, especially Katahiro, claim the three essentials are the most important in mathematics.

Section 1 "Symbols and Figures" is divided into four subsections: (abstract) symbol (抽) 象 (Problems 1 – 6), (concrete) symbol (表) 象 (Problems 7 – 11), planar figure 平形 (Problems 12 – 16), and solid figure 立形 (Problems 17 –21). Section 1 starts with the following statement:

> A symbol is not yet clarified; a figure is already clarified. They are composed of two kinds respectively[6]. As come Spring and Autumn, a theory of waxing and

[2]夫象形者, 万事之本, 為題問之首, 而常有定法之式, 亦有臨場之機, 然満干変化之道備, 而数能致其用矣.
[3]此三者, 為衆理当窮之要也.
[4]蓋自問題、答術之技, 以至天地之運、万物之気与動作云為之事, 悉莫不以具其理, 包其数焉.
[5]是以学者宜尽物変, 而窮其理矣.
[6]象者, 未顕之称; 形者, 已顕之称. 其所成各有二焉.

waning of the moon is clarified; the universe is naturally equipped with a shape of square and circle. The market price is used for everyday life and the container shape is used as a name of figures[7]. Of all theories and all things, each symbol and each figure are equipped with name. All the quantities like the length of a measure, the weight of a scale, the capacity of a container are counted by numbers according to the thing[8].

There are two kinds of symbols. Those which have no shape or those which have a shape but cannot be expressed by geometrical figure are called [abstract] symbols; those which can be compared in length and those which are represented by a numerical table are called [concrete] symbols[9].

There are two kinds of figures. Those with length and breadth are called planar figures and those with length, breadth and height are called solid figure[10]. As a symbol has only the general sum and cannot be used alone; it is used along with other things or by being applied to other things. Therefore, there is a sum, local numbers and global numbers [11]. ([double lined] The global numbers are equivalent to giving general sums. They can be given in the problem or in the procedure. These numbers are determined earlier or later according totheir quality.)[12] The symbol has its theoretical meaning and some condition gives rise to a strange symbol[13]. Each figure has a shape and according to the width or the length it can be used alone. Therefore, it is equipped with parts of the figure and the area/volume. But if we intersect, insert, manipulate or assemble these objects, a strange figure appears[14]. This is the reason why we discern symbol and figures before solving a problem; there are multitude of variations[15].

The last paragraph on geometrical figures are easy to understand, as figures 形 were classified into two subcategories: planar figures 平形 and solid figures 立形.

figures 形	planar figures 平形
	solid figures 立形

Mathematical objects other than figures are called symbols. Symbols 象 were classified into two subcategories: ( ) symbol □象 and ( ) symbol ■象. In the original text □ and ■ are hiatuses as the Katahiro could not find suitable characters to express his ideas. Following H. Komatsu, we shall read these hiatuses as (abstract) symbol (抽) 象 and (concrete) symbol (表) 象.

symbols 象	(abstract) symbols (抽) 象
	(concrete) symbols (表) 象

Section 2 is named "Flow and Ebb" and contains Problems 22 – 37. Katahiro considers here parameters in a mathematical problem. He cannot consider several parameters simultaneously but consider each parameter one by one, which makes this section hard to understand. He says

[7]如生春秋盈虧之理、顕天地方円之状者, 本自然而所具也. 如成商価日用之功、制器用什物之状者, 皆人為之所定也.

[8]衆理万物之所分, 一象一形, 各其名具, 而度長短、秤軽重、量容受、計名目者, 皆応物而自主其数也.

[9]象有二義焉. 本無状者, 雖有状、不用画図者, 謂之□; 比長短之形、成行伍之図者, 謂之□也.

[10]形有二義焉. 縦横二画, 謂之平; 縦横高三画, 謂之立也.

[11]凡象者, 毎名皆一偏之総数, 而不能自為用. 是以或托事而特為用, 或宛物而相為用. 故有通計及属一与属衆之数.

[12](乃属衆者, 与総数雖其理相同, 或題中言之, 或術中得之, 則各其数自有多少而新旧之意異矣.)

[13]其理各本自具, 而唯依所言之巧, 異象生焉.

[14]形者, 毎名有状, 拠其広狭長短自為用, 故縦横斜囲之号及計積之数相具, 然或截之, 或接之, 或容之, 或載之, 或繞之, 則随其巧, 奇形生焉.

[15]是此所以象形為題首, 而其変化無窮也.

a parameter waxes and wanes. It is very important to understand the range of a parameter and limits of the range. He also considers the cases where the parameter goes beyond the limit. In sum, Katahiro claims there are the following six statuses:

Ordinary Flow 満全	Extreme Flow 満極	Excessive Flow 満背
Ordinary Ebb 干全	Extreme Ebb 干極	Excessive Ebb 干背

Here a flow is an increasing parameter and an ebb a decreasing parameter. The parameters in a general position are called ordinary; if the parameters are on the rim of their physical existence, they are called extreme. If they represent no physical existence, they are called excessive. Katahiro tried to explain these ideas using examples given in 16 problems.

Section 3 is named "Numbers" and divided into two subsections: the first subsection is named "Dynamic and Static Numbers" 動静 and contains Problems 38 – 47.

Numbers 数	Dynamic 動
	Static 静

The second subsection deals with two kinds of well posed numbers 整数二等, that is, ordinary numbers 全 (Problems 48 – 52) and complicated numbers 繁 (Problems 53 – 57), and two kinds of inexhaustible numbers 不尽二等, that is, residual numbers 畸 (Problems 58 – 62) and degraded numbers 零 (Problems 63 – 67).

Numbers 数	Well posed 整 Rational numbers	Ordinary 全 Entire Numbers
		Complicated 繁 General fractions
	Inexhaustible 不尽 Irrational numbers	Residual 畸 Algebraic numbers
		Degraded 零 Non-algebraic Numbers

Looking at this classification of numbers, we are tempted to claim that Takebe recognized transcendental numbers. In fact, the system of real numbers is a completely modern notion. What Takebe realized was in some examples some numbers could neither satisfy any algebraic equation nor determined exactly.

3 The *Tetsujutsu Sankei*

The Mathematical Treatise on the Technique of Linkage (Technique of Linkage, for short) 綴術算経, *Tetsujutsu Sankei* is a classical Japanese mathematical text written by Takebe Katahiro in 1722 and dedicated to the 8th shōgun, Tokugawa Yoshimune. Our English translation has appeared as [5], while [9] contains an English translation of Fukyu's Technique of Linkage, another version of the Technique of Linkage.) In this treatise, Katahiro presents his most notable mathematical achievements, including, for example, an efficient calculation of π up to 42 digits and three expansion formulas for circular arc length in terms of the sagitta (maximum separation between the arc and its chord). His method for calculating is equivalent to the modern Romberg method which employs repeated Richardson extrapolation. One of the expansion formulas for arc length coincides with the Taylor expansion of the trigonometric function $(\arcsin x)^2$ in x at $x = 0$. (See [4] and [6].)

Although Takebe's book contains outstanding results of other early 18th century Japanese mathematicians, the main purpose of the Technique of Linkage is to present the author's personal view on mathematics and mathematical research. According to Katahiro, there are three aims in mathematical research, namely, rules, procedures and numbers, and two methods to reach these aims, i.e., by principles and by numbers. To illustrate his idea he employs twelve examples, including the above mentioned calculation of and the three formulas of arc length. Since it was a rare occasion for a mathematician of the Edo period to express his philosophy on mathematical research, the Technique of Linkage has for generations attracted the interest of many Japanese mathematicians.

One Chapter on a Theory of Proper Character We are at peace when we follow the spirit of mathematics. We are in trouble when we do not follow it[16]. To follow the spirit is to follow its character[17]. If we follow it, acknowledging that we will obtain a solution even before we understand [the problem], we are at peace without any doubt. Because we are at peace, we always proceed and do not stagnate. Because we always proceed and do not stagnate, there is nothing which cannot be accomplished[18]. If we do not follow it, then without knowing if we will be able to obtain [a solution] or not before we understand [the problem], we are in doubt[19]. Because we are in doubt, we suffer and are daunted[20]. Because we suffer and are daunted, it is difficult to obtain [a solution][21]. After I [myself] started to learn mathematics, looking for the easy way I was suffering from mathematical rules for a long time[22]. Certainly, this was because I did not exhaust my own character[23]. Gradually after 60 days' struggle, I could realize my born character was distorted and became convinced that I should follow the spirit of mathematics[24].

Alas, our own born character, straight or distorted, is native, we cannot change it. Even if we study hard, it cannot be improved; even if we forget and abandon it, it cannot be damaged in the least[25]. That is, we should speculate about its distortion but we should not speculate about its straightness[26]. If we do not exhaust our own character, we cannot understand the truth which follows the character of mathematics[27]. But many people do not understand the it is natural that the native

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16 自質の説 [1] 算の数の心に従うときは、泰し。[2] 従わざるときは、苦しむ。

17 [3] 所謂、心に従うは即ち質に従うなり。

18 [4] 其の従う所以は其の事未だ会せざる以前に必ず得べきを肯ずる心有るゆえ、疑うこと無くして泰きに居る。[5] 泰きに居るゆえ、常に為して止まず。[6] 常に為して止まざるゆえ、成し得ずと云うことなし。

19 [7] 従わざる者は、其の事未だ会せざる以前に、得べきをも得べからざるをも料ること無くして疑う。

20 [8] 疑うゆえに、苦しみ屈す。

21 [9] 苦しみ屈するゆえ、成し得ること難し。

22 [10] (吾) 算を学びてより常に安行ならんことを意うて算法に苦しむこと久し。

23 [11] 蓋し是、未だ自己の質分を尽くさざるゆえなり。

24 [12] 徐く六旬に及ばんとする比、自ら生れ得る本質の偏駁なることを実に識り得て算の数の質に従うことを肯ぜり。

25 [13] 嗚呼、自己の粋偏の本質は人々生れ得る侭にして、学び尽くすと雖も更に増長すること無く、又、廃忘すと雖も些も損消すること靡し。

26 [14] 乃ち其の偏質をば思議[左傍訓：おもい、はかる。] すべし。[15] 粋質をば思議すべからず。

27 [16] 人々自ら此の質分を尽くさざらんは敢て算の質に従う真実を会すべからず。

character may be straight or distorted[28]. Instead, they think that everything becomes clear after complete study and that it is not necessary to use force. How misled they are![29] These people think that one can obtain the straight character by study[30]. How can such study change the [person's] character [into one which is] purely straight?[31]

Certainly, even if, exhausting our own character, we embody the Way [of Mathematics], the native character is the native character; it does not move, does not change. Also, there is nothing to be puzzled and nothing to be clarified. At any time when we are given a problem, following its difficulty, we cannot be away from using force[32].

Also, once I heard that one person swallowed his art. Does this refer to the person whose character is purely straight?[33]Deliberating about him, when I make the art follow me and enter into my heart, although what can be planned follows me, what cannot be planned may not follow me; this is because there is a difference between what can be planned and what cannot be planned[34]. I declare that, when I immerse myself completely in mathematics without any resistance, I [myself] and the Way [of Mathematics] become mixed together, what can be planned follows me as what can be planned and what cannot be planned also follows me as what cannot be planned[35]. This is one outcome of the embodiment of the Way[36]. If one knows the Way of Mathematics in heart and explain it in words, he is dishonest[37]. If one embodies the Way and proceeds [in mathematics], he is [honest] in the truth[38]. We cannot speculate about the truth of the embodiment of the Way[39]. But in training myself in this truth which should not be speculated, I [myself] am sure there is one rule which concerns the native character[40]. But I [myself] am not yet mature in the Way. Therefore, I dare not explain it[41]. When I become confident about its

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[28][17] 然るに人皆質分の粋偏生得の自然たることを暁さず。

[29][18] 学び尽くして後は咸く通明にして力を用いること無しと為り。[19] 惑える哉。

[30][20] 此の如きは純粋の質は学びて得る者と思える也。

[31][21] 如何ぞ学びて純粋の質に変成すること有らんや。

[32][22] 蓋し其の質分を尽くし道に体するとも、生得の質は便ち生得の質にして、動くこと無く、変ずること無く、亦、惑うべきことも無く、還明かなるべきことも無く、而も毎に事に臨みては難易に従いて力を用いずと云うこと無き耳。

[33][23] 亦、嘗て聞けり、或其の芸を呑むと。[24] 是は此本質の純粋なる者を謂う歟。

[34][25] 熟思うに、芸を以って己に従えて自心の中に容るるときは、議るべきと議るべからざるとの分有るゆえ、其の議るべき限りは我に従うと雖も、議るべからざるに到りては我に従わざること有り。

[35][26] (吾) は謂う、自己を以って些も忤うこと無く、咸く算の中へ入るときは、自心と道と混一にして議るべきは議るべくして我に従い、議るべからざるは議るべからずして又我に従う。

[36][27] 是、乃ち道に体するの一端也。

[37][28] 夫れ、算の道を心に知りて言に説く者は不実なり。

[38][29] 道に体して事に行う者は真実也。

[39][30] 此の道に体する真実は敢て思議すべからざる者也。

[40][31] 而るに其の思議すべからざる真実に於いて自ら是を修するに (吾) 生得の質に随う一个の則有ることを肯じ得たり。

[41][32] 然れども (吾) 道猶、未だ熟せず。[33] 故に、之を説かざる也。

meaning, I will explain it. This is indeed my distorted character[42].

Certainly, if I were of purely straight character, I would have no intention to explain a single word about it. Why should I explain?[43] What is to be explained is that the native character is distorted[44].

Generally speaking, the character is not equal among people; it may be straight or distorted, warm or cold[45]. It is indeed in this way that I [myself] follow the character of mathematics. But it is not always like this that others also follow it[46]. Therefore, when a student of mathematics looks at this book, he should not take it [as being] right immediately; he should not take it [as being] wrong without thinking[47]. I would like to explain the reason why one can recognize one's own native character and that the truth of mathematics follows the character[48].

4 Meiji Restoration

The Meiji Restoration of 1868 was a turning point in the history of Japan from the feudalism to the constitutional monarchism. The "ordinance on school system" (1872) of the new government defined mathematics in Japanese schools to be of "European style" thus abandoning the Japanese traditional mathematics, *wasan.* This policy was proved to be efficient at least for a half century and Takebe's philosophy on Mathematics was buried in the complete oblivion.

In 1896, Endō Toshisada (1843 - 1915) wrote the *History of Mathematics in Great Japan* [1], which was the first monograph on traditional Mathematics in Japan, with many patriotic expressions to claim the Japanese identity. This book was re-edited by Mikami Yoshio (1875 - 1950), corrected by Hirayama Akira (1904 - 1998) an republished in 1960. In Endo's book, "Three Essentials" were cited only as the name of a chapter wigh no explanation, while the "circle theory" in *wasan* was described in details as one of the achivements which could emulate the European counterpart.

In 1954 the *History of Mathematics in Japan before the Meiji Restoration* was published by Japan Academy. The true author of this five volume monograph was Fujiwara Matsusaburo (1881 - 1946). He wrote at [3], p.385 that Volume Four of the *Taisei Sankei* is "very strange and meaningless as mathematical theory." Because of this negative comments almost no research on Volume 4 had been done until Xu Zelin published [10] in 2002. In this important article he examined the Three Essentials and understood it in the context of traditional Chinese culture. Recently, there have appeared a few papers like Ozaki Fumiaki [7] and Komatsu Hikosaburo [2].

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[42] [34]其の言うべきを肯じて後に言うこと有らん歟。[35]是、即ち(吾が)偏質也。

[43] [36]蓋し純粋の質にしては総て一字として説くべきこと無し。[37]何をか説くこと有らん。

[44] [38]其の説くこと有るは即ち是、生得の偏質を説く者也。

[45] [39]凡そ生得粋偏厚薄の質、人人斉しき者有ること無し。

[46] [40]是を以って(吾)、算の質に従う所以を説くこと正に此の如しと雖も、人も亦、質に従う所以は是の如しと云うに非ず。

[47] [41]故に、如し算を学ぶ者此の説を聴きて徒に是とすること無かれ。[42]又空しく非とすること無かれ。

[48] [43]唯人人自己の生れ得る質を実に識り得て、質に従いて算の数の真実、質に従う所以を説くべき也。

Closing Remark

We can say that mathematicians in the Meiji period were ignorant of the mathematical philosophy like "Three Essentials", which stemmed from Chinese tradition. They did not endeavor to investigate the Taisei Sankei to find ghe Chinese influences. Instead they were keen to compare the achievements of the "circle theory" with teh contenporary European mathematics. Some of the achievements of *wasan* could indeed emulate those in Europe. People were also interested in the psychological rivalry between Takebe Katahiro and his master Seki Takakazu, as describeed in the *Tetsujutsu Sankei*.

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